IOSR Journal of Engineering (IOSRJEN)
ISSN: 2250-3021 Volume 2, Issue 6 (June 2012), PP 1420-1423
www.iosrjen.org

A STUDY ON ANTI L-FUZZY SUBHEMIRINGS OF A HEMIRING

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ABSTRACT**:** *In this paper, we made an attempt to study the algebraic nature of an anti L-fuzzy subhemiring of a hemiring .2000 AMS Subject classification: 03F55, 06D72, 08A72.*

KEY WORDS**:** *L-fuzzy set, anti L-fuzzy subhemiring, pseudo anti L-fuzzy coset.*

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (*R ; + ; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras (*R ; + ; .*) share the same properties as a ring except that (*R ; +*) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh[12], several researchers explored on the generalization of the concept of fuzzy sets. The notion of anti fuzzy Left h- ideals in Hemirings was introduced by Akram.M and K.H.Dar [1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan[6]. In this paper, we introduce the some Theorems in anti L-fuzzy subhemiring of a hemiring.

1.PRELIMINARIES:

1.1 Definition: Let X be a non-empty set and L = (L, $\leq$) be a lattice with least element 0 and greatest element 1. A **L-fuzzy subset** A of X is a function A : X $\rightarrow$ L.

1.2 Definition: Let (R , + , .) be a hemiring. A L-fuzzy subset A of R is said to be an anti L-fuzzy subhemiring (ALFSHR) of R if it satisfies the following conditions:

(i) $\mu_A(x+y) \leq \mu_A(x) \vee \mu_A(y)$,

(ii) $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in R.

1.3 Definition: Let A and B be L-fuzzy subsets of sets G and H, respectively. The anti-product of A and B, denoted by AxB, is defined as AxB $=\{\langle (x, y), \mu_{AxB}(x,y) \rangle$ / for all x in G and y in H $\}$, where $\mu_{AxB}(x, y) = \mu_A(x) \vee \mu_B(y)$.

1.4 Definition: Let A be a L-fuzzy subset in a set S, the anti-strongest L-fuzzy relation on S, that is a L-fuzzy relation on A is V given by $\mu_V(x, y) = \mu_A(x) \vee \mu_A(y)$, for all x and y in S.

1.5 Definition: Let (R, +, .) and (R^l, +, .) be any two hemirings. Let f : R $\rightarrow$ R^l be any function and A be an anti L-fuzzy subhemiring in R, V be an anti L-fuzzy subhemiring in f (R)= R^l, defined by $\mu_V(y) = \inf_{x \in f^{-1}(y)} \mu_A(x)$, for all x in R and y in R^l. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.6 Definition: Let A be an anti L-fuzzy subhemiring of a hemiring (R, +, .) and a in R. Then the pseudo anti L-fuzzy coset $(aA)^p$ is defined by $((a\mu_A)^p)(x) = p(a)\mu_A(x)$, for every x in R and for some p in P.

2. PROPERTIES OF ANTI L-FUZZY SUBHEMIRING OF A HEMIRING

2.1 Theorem: Union of any two anti L-fuzzy subhemiring of a hemiring R is an anti L-fuzzy subhemiring of R.

Proof: Let A and B be any two anti L-fuzzy subhemirings of a hemiring R and x and y in R. Let A$=\{(x, \mu_A(x)) / x\in R\}$ and B$=\{(x, \mu_B(x)) / x\in R\}$ and also let C =A$\cup$B $=\{ (x, \mu_C(x)) / x\in R\}$, where $\mu_A(x) \vee \mu_B(x)= \mu_C(x)$. Now, $\mu_C(x+y) \leq \{\mu_A(x)\vee\mu_A(y)\}\vee \{ \mu_B(x)\vee\mu_B(y)\}= \mu_C(x)\vee\mu_C(y)$. Therefore, $\mu_C(x+y) \leq$

$\mu_C(x)\vee\mu_C(y)$, for all x and y in R. And, $\mu_C(xy) \leq \{ \mu_A(x) \vee \mu_A(y) \}\vee\{ \mu_B(x) \vee \mu_B(y)\}= \mu_C(x) \vee \mu_C(y)$. Therefore, $\mu_C(xy) \leq \mu_C(x) \vee \mu_C(y)$, for all x and y in R. Therefore C is an anti L-fuzzy subhemiring of a hemiring R.

2.2 Theorem: The union of a family of anti L-fuzzy subhemirings of hemiring R is an anti L-fuzzy subhemiring of R.

Proof: It is trivial.

2.3 Theorem: If A and B are any two anti L-fuzzy subhemirings of the hemirings R_1 and R_2 respectively, then anti-product AxB is an anti L-fuzzy subhemiring of R_1xR_2.

Proof: Let A and B be two anti L-fuzzy subhemirings of the hemirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in R_1xR_2. Now, $\mu_{AxB} [(x_1, y_1)+ (x_2, y_2)] \leq \{ \mu_A(x_1) \vee \mu_A(x_2) \}\vee\{ \mu_B(y_1) \vee \mu_B(y_2) \} = \mu_{AxB} (x_1, y_1) \vee\mu_{AxB} (x_2, y_2)$. Therefore, $\mu_{AxB} [(x_1, y_1) +(x_2, y_2)] \leq \mu_{AxB} (x_1, y_1) \vee \mu_{AxB} (x_2, y_2)$. Also, $\mu_{AxB} [(x_1, y_1)(x_2, y_2)] \leq \{\mu_A(x_1) \vee \mu_A(x_2)\} \vee\{ \mu_B(y_1) \vee \mu_B(y_2)\} = \mu_{AxB} (x_1, y_1) \vee \mu_{AxB} (x_2, y_2)$. Therefore, $\mu_{AxB} [(x_1, y_1)(x_2, y_2)] \leq \mu_{AxB} (x_1, y_1) \vee \mu_{AxB} (x_2, y_2)$. Hence AxB is an anti L-fuzzy subhemiring of hemiring of R_1xR_2.

2.4 Theorem: Let A be a L-fuzzy subset of a hemiring R and V be the anti-strongest L-fuzzy relation of R. Then A is an anti L-fuzzy subhemiring of R if and only if V is an anti L-fuzzy subhemiring of RxR.

Proof: Suppose that A is an anti L-fuzzy subhemiring of a hemiring R. Then for any $x=(x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR. We have, $\mu_V (x+y) = \mu_A(x_1+ y_1) \vee \mu_A(x_2+y_2) \leq \{\mu_A(x_1)\vee\mu_A(y_1)\}\vee\{\mu_A(x_2) \vee \mu_A(y_2)\}= \mu_V (x_1, x_2)\vee \mu_V (y_1, y_2) = \mu_V (x) \vee \mu_V (y)$. Therefore, $\mu_V (x + y) \leq \mu_V (x) \vee \mu_V (y)$, for all x and y in RxR. And, $\mu_V(xy) = \mu_A(x_1y_1)\vee\mu_A(x_2y_2) \leq\{\mu_A(x_1)\vee\mu_A(y_1)\}\vee\{ \mu_A(x_2) \vee \mu_A(y_2)\}= \mu_V (x_1, x_2) \vee \mu_V (y_1, y_2) = \mu_V (x) \vee \mu_V (y)$. Therefore, $\mu_V (xy) \leq \mu_V (x) \vee \mu_V (y)$, for all x and y in RxR. This proves that V is an anti L-fuzzy subhemiring of RxR. Conversely assume that V is an anti L-fuzzy subhemiring of R x R, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR, we have $\mu_A(x_1+y_1)\vee\mu_A(x_2+y_2) =\mu_V (x+y)\leq \mu_V(x)\vee\mu_V(y) = \mu_V(x_1, x_2)\vee\mu_V(y_1, y_2) = \{\mu_A(x_1)\vee\mu_A(x_2)\}\vee\{\mu_A(y_1) \vee \mu_A(y_2)\}$. If $x_2 =0$, $y_2 = 0$, we get, $\mu_A(x_1+ y_1) \leq \mu_A(x_1) \vee \mu_A(y_1)$, for all x_1 and y_1 in R. And, $\mu_A(x_1y_1) \vee \mu_A(x_2y_2) =\mu_V (xy) \leq \mu_V (x)\vee\mu_V (y) = \mu_V (x_1, x_2) \vee \mu_V (y_1, y_2) =\{\mu_A(x_1)\vee\mu_A(x_2) \}\vee\{\mu_A(y_1) \vee \mu_A(y_2)\}$. If $x_2 = 0$, $y_2 = 0$, we get $\mu_A(x_1y_1) \leq \mu_A(x_1) \vee \mu_A(y_1)$, for all x_1 and y_1 in R. Therefore A is an anti L-fuzzy subhemiring of R.

2.5 Theorem: A is an anti L-fuzzy subhemiring of a hemiring (R, +, .) if and only if $\mu_A(x+y) \leq \mu_A(x) \vee \mu_A(y)$, $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in R.

Proof: It is trivial.

2.6 Theorem: If A is an anti L-fuzzy subhemiring of a hemiring (R, +, .), then $H = \{ x / x\in R: \mu_A(x) = 0\}$ is either empty or is a subhemiring of R.

Proof: It is trivial.

2.7 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring (R, +, .). If $\mu_A(x+ y) = 1$, then either $\mu_A(x) =1$ or $\mu_A(y) = 1$, for all x and y in R.

Proof: It is trivial.

2.8 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring (R, +, .), then the pseudo anti L-fuzzy coset $(aA)^p$ is an anti L-fuzzy subhemiring of a hemiring R, for every a in R.

Proof: Let A be an anti L-fuzzy subhemiring of a hemiring R. For every x and y in R, we have, $((a\mu_A)^p)(x+y) \leq p(a)\{\mu_A(x)\vee\mu_A(y)\}= p(a)\mu_A(x)\vee p(a)\mu_A(y) = ((a\mu_A)^p)(x) \vee((a\mu_A)^p)(y)$. Therefore, $((a\mu_A)^p)(x+y)\leq((a\mu_A)^p)(x)\vee((a\mu_A)^p)(y)$. Now, $((a\mu_A)^p)(xy) \leq p(a)\{ \mu_A(x) \vee\mu_A(y)\}= p(a)\mu_A(x)\vee p(a)\mu_A(y) = ((a\mu_A)^p)(x)\vee((a\mu_A)^p)(y)$. Therefore, $((a\mu_A)^p)(xy) \leq ((a\mu_A)^p)(x) \vee ((a\mu_A)^p)(y)$. Hence $(aA)^p$ is an anti L-fuzzy subhemiring of a hemiring R.

2.9 Theorem: Let (R, + , .) and (R^I, +, .) be any two hemirings. The homomorphic image of an anti L-fuzzy subhemiring of R is an anti L-fuzzy subhemiring of R^I.

Proof: Let $f : R \to R^I$ be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let V = f(A), where A is an anti L-fuzzy subhemiring of R. Now, for f(x), f(y) in R^I, $\mu_v(f(x)+f(y)) \leq \mu_A(x+y) \leq \mu_A(x)\vee\mu_A(y)$, which implies that $\mu_v(f(x) + f(y)) \leq \mu_v(f(x)) \vee \mu_v(f(y))$. Again, $\mu_v(f(x)f(y)) \leq \mu_A(xy) \leq \mu_A(x)\vee\mu_A(y)$, which implies that $\mu_v(f(x)f(y)) \leq \mu_v(f(x)) \vee \mu_v(f(y))$. Hence V is an anti L-fuzzy subhemiring of R^I.

2.10 Theorem: Let (R, +, .) and (R^I, +, .) be any two hemirings. The homomorphic preimage of an anti L-fuzzy subhemiring of R^I is an anti L-fuzzy subhemiring of R.

Proof: Let V = f(A), where V is an anti L-fuzzy subhemiring of R^I. Let x and y in R. Then, $\mu_A(x+y)= \mu_v(f(x+y)) \leq \mu_v(f(x)) \vee\mu_v(f(y)) = \mu_A(x)\vee \mu_A(y)$, which implies that $\mu_A(x+y) \leq \mu_A(x)\vee\mu_A(y)$. Again, $\mu_A(xy)=\mu_v(f(xy)) \leq \mu_v(f(x)) \vee \mu_v(f(y)) = \mu_A(x)\vee\mu_A(y)$
which implies that $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$. Hence A is an anti L-fuzzy subhemiring of R.

2.11 Theorem: Let (R, +, .) and (R^I, +, .) be any two hemirings. The anti-homomorphic image of an anti L-fuzzy subhemiring of R is an anti L-fuzzy subhemiring of R^I.

Proof: Let f : R $\to$ R^{l} be an anti-homomorphism. Then, f(x+y) = f(y) + f(x) and f(xy) = f(y)f(x), for all x and y in R. Let V = f(A), where A is an anti L-fuzzy subhemiring of R. Now, for f(x), f(y) in R^{l}, μ_v(f(x)+f(y)) $\leq$ μ_A(y+x) $\leq$ μ_A(y)$\vee$ μ_A(x) = μ_A(x)$\vee$ μ_A(y), which implies that μ_v(f(x)+f(y)) $\leq$ μ_v(f(x)) $\vee$ μ_v(f(y)). Again, μ_v(f(x)f(y)) $\leq$ μ_A(yx) $\leq$ μ_A(y) $\vee\mu_A$(x) = μ_A(x)$\vee\mu_A$(y), which implies that μ_v(f(x)f(y)) $\leq$ μ_v(f(x)) $\vee$ μ_v(f(y)). Hence V is an anti L-fuzzy subhemiring of R^{l}.

2.12 Theorem: Let (R, + , .) and (R^{l}, +, .) be any two hemirings. The anti-homomorphic preimage of an anti L-fuzzy subhemiring of R^{l} is an anti L-fuzzy subhemiring of R.

Proof: Let V = f(A), where V is an anti L-fuzzy subhemiring of R^{l}. Let x and y in R. Then, μ_A(x+y) = μ_v(f(x+y)) $\leq$ μ_v(f(y))$\vee$ μ_v(f(x)) = μ_A(x) $\vee\mu_A$(y), which implies that μ_A(x + y) $\leq$ μ_A(x) $\vee$ μ_A(y). Again, μ_A(xy) = μ_v(f(xy)) $\leq$ μ_v(f(y)) $\vee$ μ_v(f(x)) = μ_A(x) $\vee\mu_A$(y), which implies that μ_A(xy) $\leq$ μ_A(x) $\vee$ μ_A(y). Hence A is an anti L-fuzzy subhemiring of R.

In the following Theorem ◦ is the composition operation of functions

2.13 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then A◦f is an anti L-fuzzy subhemiring of R.

Proof: Let x and y in R.Then we have, (μ_A◦f)(x+y) = μ_A(f(x)+ f(y))$\leq$ μ_A(f(x))$\vee$ μ_A(f(y)) $\leq$ (μ_A◦f)(x) $\vee$ (μ_A◦f)(y), which implies that (μ_A◦f)(x+y) $\leq$ (μ_A◦f)(x) $\vee$ (μ_A◦f)(y). And, (μ_A◦f)(xy) = μ_A (f(x)f(y)) $\leq$ μ_A(f(x)) $\vee$ μ_A(f(y)) $\leq$ (μ_A◦f)(x) $\vee$(μ_A◦f)(y), which implies that (μ_A◦f)(xy) $\leq$ (μ_A◦f)(x) $\vee$ (μ_A◦f)(y). Therefore A◦f is an anti L-fuzzy subhemiring of a hemiring R.

2.14 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. Then A◦f is an anti L-fuzzy subhemiring of R.

Proof: Let x and y in R. Then we have, (μ_A◦f)(x+y) = μ_A(f(y)+f(x)) $\leq$ μ_A(f(x)) $\vee$ μ_A(f(y)) $\leq$ (μ_A◦f)(x) $\vee$ (μ_A◦f)(y), which implies that (μ_A◦f)(x+y) $\leq$ (μ_A◦f)(x) $\vee$ (μ_A◦f)(y). And (μ_A◦f)(xy) = μ_A(f(y)f(x))$\leq$ μ_A(f(x)) $\vee\mu_A$(f(y))$\leq$ (μ_A◦f)(x) $\vee$ (μ_A◦f)(y), which implies that (μ_A◦f)(xy) $\leq$ (μ_A◦f)(x) $\vee$(μ_A◦f)(y). Therefore A◦f is an anti L-fuzzy subhemiring of a hemiring R.

2.15 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring R, A^{+} be a L-fuzzy set in R defined by A^{+}(x) = A(x) +1 – A(0), for all x in R . Then A^{+} is an anti L-fuzzy subhemiring of a hemiring R.

Proof : Let x and y in R. We have, A^{+}(x+y) = A(x+y) +1- A(0) $\leq$ {A(x)$\vee$A(y)}+1- A(0) = A^{+}(x) $\vee$ A^{+}(y). Therefore, A^{+}(x+y) $\leq$ A^{+}(x) $\vee$ A^{+}(y), for all x, y in R. Similarly, A^{+}(xy) = A(xy) +1- A(0) $\leq$ {A(x)$\vee$A(y)}+1- A(0) = A^{+}(x) $\vee$ A^{+}(y). Therefore, A^{+}(xy) $\leq$ A^{+}(x) $\vee$ A^{+}(y), for all x, y in R. Hence A^{+} is an anti L-fuzzy subhemiring of a hemiring R.

2.16 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring R, A^{+} be a L-fuzzy set in R defined by A^{+}(x) = A(x) +1 – A(0), for all x in R. Then there exists 0 in R such that A(0) = 1 if and only if A^{+}(x) = A(x).

Proof : It is trivial.

2.17 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring R, A^{+} be a L-fuzzy set in R defined by A^{+}(x) = A(x) +1 – A(0), for all x in R. Then there exists x in R such that A^{+}(x) = 1 if and only if x = 0.

Proof: It is trivial.

2.18 Theorem : Let A be an anti L-fuzzy subhemiring of a hemiring R, A^{+} be a L-fuzzy set in R defined by A^{+}(x) = A(x) +1 – A(0), for all x in R. Then (A^{+})$^{+}$ = A^{+}.

Proof: It is trivial.

2.19 Theorem: Let A be an anti L-fuzzy subhemiring of a hemiring R, A^{0} be a L-fuzzy set in R defined by A^{0}(x) = A(0)A(x), for all x in R.. Then A^{0} is an anti L-fuzzy subhemiring of the hemiring R.

Proof: For any x in R, we have A^{0}(x+y) = A(0)A(x+y) $\leq$ A(0){ A(x) $\vee$ A(y) }= A(0)A(x) $\vee$A(0)A(y) = A^{0}(x) $\vee$ A^{0}(y). That is A^{0}(x+y) $\leq$ A^{0}(x) $\vee$ A^{0}(y), for all x, y in R. Similarly, A^{0}(xy) = A(0)A(xy) $\leq$ A(0){A(x) $\vee$A(y)}= A(0)A(x) $\vee$A(0)A(y) = A^{0}(x) $\vee$ A^{0}(y). That is A^{0}(xy) $\leq$ A^{0}(x) $\vee$ A^{0}(y), for all x, y in R. Hence A^{0} is an anti L-fuzzy subhemiring of the hemiring R.

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